Multi-Aircraft Routing and Traffic Flow Management under Uncertainty*

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Abstract

A major portion of the delay in the Air Traffic Management Systems (ATMS) in US and Europe arises from the convective weather. In the current practice of managing air traffic, the predicted storm zones are considered as deterministic obstacles and hence they are completely avoided. As a result, the current strategy is too conservative and incurs a high delay. In reality, the dynamics of the convective weather is stochastic in nature. Hence, the capacity of the airspace is probabilistic, which reduces drastically with the convective weather. Our research objective is to deal with the dynamic and stochastic nature of the storms and add recourse in the routing and the flow management problem. We address the *multi-aircraft* flow management problem using a stochastic dynamic programming algorithm, where the evolution of the weather is modelled as a stationary Markov chain. Our solution provides a dynamic routing strategy for *"N-aircraft"* that minimizes the *expected delay* of the overall system while taking into consideration of the constraints obtained by *the sector capacities*, as well as avoidance of *conflicts* among the aircraft. Our simulation suggests that a significant improvement in delay can be obtained by using our methods over the existing methods.

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1 Introduction

The air traffic management systems in US and Europe are approaching a critical saturation level. There has been a steady increase in delays over the last decade. Regular disruptions such as weather cause an explosion of flight delays and cancellations. In figure 1, we see that the number of delayed flights due to convective weather has been increasing since the year 1995 [5]. There is a significant number of delayed flights all year long, with particularly high number of delays in the summer months. The airspace capacity reduces drastically with the presence of convective weather. The drastic reduction of airspace capacity interrupts traffic flows and causes delays that ripple through the system. Consequently, weather related delays contribute to around 80% of the total year in most of the years in US since 1995. Though the years 2001 and 2002 have been better (in terms of delays) than the year 2000 due to weaker economy, the situation is expected to get worse in the coming years. Moreover, these delays do not depict the entire picture of the situation, as cancelled flights are not included in the figures. According to FAA, the cancellations have increased by 67% since 1995. The cost of delays to airlines and passengers are billions of US dollars per year.

According to FAA 2002, the main sources that contribute to the delays in the US are weather, equipment, volume, and runways. Delays caused by weather are dynamic and stochastic in nature. Other sources cause delays which can be modelled in a deterministic framework. There has been a major effort to address delay in the traffic flow management problem in the deterministic setting [1, 4, 8, 3, 6, 7, 9], where demand and capacities are considered deterministic. In these works, various traffic flow management algorithms are proposed

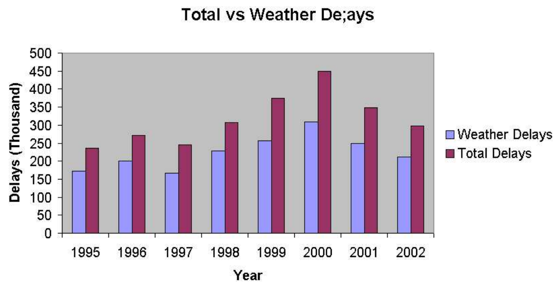

Figure 1: Number of total delayed flights vs delayed flights due to weather(in thousands) by year. Source: FAA OPSNET data.

in order to reduce the system delay, given that the system capacity is exactly known. However, the major contributor of delay is weather, which is probabilistic in nature and cannot be addressed in this framework. In the National Airspace System, Federal Aviation Administration controllers and airline dispatchers make Traffic flow management (TFM) and routing decisions 3-4 hours in advance of the actual operation. Knowledge of the location and the intensity of the hazardous convective weather 3-4 hours ahead is key to select air routes. The rerouting and the redistribution strategies of the traffic affected by convective weather is discussed in recent works [5, 13]. The main focuss of these works has been to manage the traffic flow, given that the convective weather has already been taken place in the airspace. In addition to these models, it is imperative to have a strategy where recourse is included in the planning process, such that we can *plan to replan* under the weather uncertainty. In our previous work [11], we incorporated recourse in the planning process where we addressed *the single aircraft* problem using Markov decision processes (where the weather processes is modelled as a stationary Markov chain) and a dynamic programming algorithm. Our approach provides a set of optimal decisions to *a single aircraft* that starts moving towards the destination along a certain path, with the recourse option of choosing a new path whenever new information is obtained, such that the expected delay is minimized. As we addressed the problem in the stochastic framework, we obtain "the best policy". In addition, we proposed an algorithm for dynamic routing where the solution is robust with respect to the estimation errors of the storm probabilities [10]. To the Bellman equations, which are derived in solving the dynamic routing strategy of an aircraft, we add a further requirements: we assume that the transition probabilities are unknown, but bounded within a convex set. Our algorithm optimizes the performance of the system, given there are errors in the estimation of the probabilities of the storms.

However, as the algorithms provide a routing strategy for a *single aircraft*, they cannot solve a real life flow management problem entirely which requires handling of multiple aircraft. Moreover, the model that we used to describe the dynamics of the weather was expressed in a binary manner, where for each predicted zone, we allowed the convective weather to either stay or not stay. This is rather simplistic and cannot capture the real life dynamic nature of the weather.

In this paper, we extend our model for *multiple aircraft*. The problem of routing under convective weather becomes much more complex in a congested airspace because both aircraft conflicts and traffic flow management issues must be resolved at the same time. In this work, we provide a dynamic routing strategy for multiple aircraft that minimizes the expected delay of the overall system while satisfying the consideration of the constraints obtained by the sector capacity, as well as avoidance of conflicts among the aircraft. Moreover, we have used a more general weather dynamic model where the predicted zones can have more than two different states.

2 Weather uncertainty model

Various weather teams (CCFP, ITWS etc) produce predictions that some zones in the airspace may be unusable in certain time interval and their predictions are dynamically updated at every $T =$ 15 minutes. The later an event is from the prediction time, the more unreliable it becomes. It is reasonable to assume that we have a deterministic knowledge of the weather in the time interval of $0-15$ minutes in future. Hence, each aircraft has a perfect knowledge about the weather in the regions that are 15 minutes (15 times the velocity of the aircraft provides the distance) away from it.

We discritize time as $1, 2, ..., n$ *stages* according to the weather update. Stage 1 corresponds to $0-15$ minutes from the current time, stage 2 corresponds to $15-30$ minutes from the current time. We choose n that accommodates the worst case routing of the aircraft. Let there is only one storm that is predicted to take place at the region K_1. In our previous work [11], we defined state "1" corresponding to the state of having a storm in a region at a particular stage and state "0" corresponding to the state of having no storm at a particular stage in that region. As we know the status of any storm in the time interval of $0-15$ minutes (stage 1), we can assign 0(1) deterministically to every storm at the stage 1. Moreover, we assumed that we knew the conditional probability of having (not having) a storm in a region in a 15 minutes time interval (stage q), given there is a storm(no storm) in the previous stage (stage $q-1$) in the region, and the dynamics is considered Markovian. The conditional probability only depends on the status of the storm in the stage p only.

However, this model cannot capture the realistic weather dynamics accurately. In figure 2, we see that the predicted convective weather vs actual weather in a typical summer day. There are different colors associated with different predicted zones which correspond to different density, i.e., the percentage of the area of the predicted zones that will be affected by the storm. The actual outcomes of the predictions are more complex than that to be described by binary states. Instead of fully realizing in the worst form or not realizing at all, it can realize in many intermediate forms.

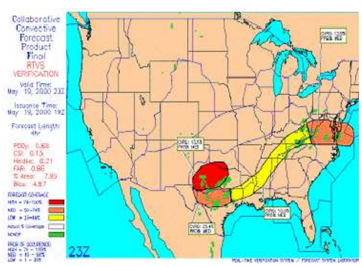

Figure 2: A CCFP weather prediction vs actual weather in a summer day (19th May, 2001).

In this paper, we propose that each predicted convective weather zone can have multiple outcomes. Depending on the coverage area and the intensity of the prediction, we allow to have different realizations. Hence, the Markov chain that we propose is not a two state one, instead it is a l state Markov chain, where l is the number of possible outcomes of the prediction in the region K_1 (figure 3). $K_{11}, K_{12}, \ldots, K_{1l}$, are the possible outcome regions in K_1 ($K_{1i} \subset K_1 \forall i < l$ and $K_{il} = K_1$). "0" corresponds to the state where there is no storm in the region k_1, "1" corresponds to the state where there is a storm, but only materialized in the region K_{11}, and so on. State "l" corresponds to the worst possible outcome when K_{il} or the whole region K_1 has been affected by the storm. We define p_{ij} as the probability of the storm state to be j in the next stage if the current state is i. If there are m predicted convective zones, the system can be represented by a state of m tuple vector, with a cardinality of $(l+1)^m$.

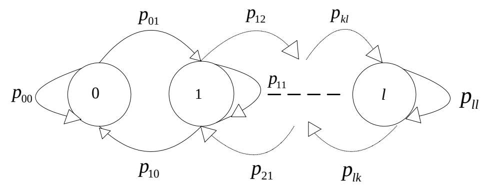

Figure 3: A 2-D view of the problem.

3 Problem Formulation

We consider a two dimensional flight plan of multiple aircraft whose nominal paths are obstructed by predicted convective weathers. All the aircraft considered here are in the TMA/En-route portion of their flights. Hence, the velocities of all the aircraft considered are constant.

We use a rectangular gridding system to represent the airspace where we consider each grid point as a way point. If any aircraft is at a point 'A' at the beginning of the stage, and the solution of the problem provides the grid point 'B' that the aircraft will reach at the end of the stage, the aircraft will fly a straight line path connecting 'A' and 'B'. There are N aircraft currently positioned at $O^1, O^2, \ldots, O^N$ and the destination points of the aircraft are $D^1, D_2, \ldots, D^N$ (Figure 4). $O^i = [O^i(x), O^i(y)]^T \in \mathbf{R}^2$, where $O^i(x)$ and $O^i(y)$ are respectively the x and y coordinates of the origin of aircraft i. Similarly, $D^i = [D^i(x), D^i(y)]^T \in \mathbf{R}^2$, where $D^i(x)$ and $D^i(y)$ are respectively the x and y coordinates of the destination of aircraft i. Without the presence of convective weather, aircraft will try to follow the straight line connecting the origin and the destination, if those paths don't result in conflicts. There is a prediction that there can be m storms located at $K_1, \ldots, K_m$ places such that those zones might be unusable at certain time. $w \in W$ are the weather states and $|W| = (l+1)^m$. The airspace that is considered here is confined in f sectors, and the capacities of the sectors are $C_1, \ldots, C_f$. The predictions are dynamically updated with time. In the current practice, these stochastic convective zones are assumed to be completely unusable, and solution proceeds as if they are deterministic constraints. As those zones were just predicted to be of unusable with a certain probability, it often turns out that the zones were perfectly usable. As the routing strategies do not use these resources, airspace resources are under-utilized, leading to congestion in the remaining airspace through ripple effect.

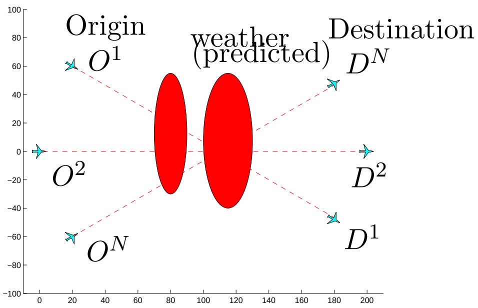

Figure 4: A 2-D view of the problem.

In our proposed model, we will not exclude the zones which are predicted to be unusable (with some probability) at a certain time and we will take into consideration the fact that there will more updates with the course of flight and recourse will be applied accordingly. We take a less conservative route in avoiding the bad weather zone where we take a risk in delay to attain a better expected delay instead of avoiding the bad weather zones deterministically. We consider the following two schemes,

1. All of the N aircraft has the equal priority.

2. Every aircraft has different priority. Without the loss of generality, we assume that (priority of aircraft 1)>(priority of aircraft 2)> $\ldots$ >(priority of aircraft N).

Scheme 1: All of the N aircraft has the equal priority.

At each stage (15 minutes time span, before the next update), the storm state is assumed to stay constant. $X_1 \in \mathbf{R}^2, X_2 \in \mathbf{R}^2, \ldots, X_N \in \mathbf{R}^2$ represent the locations of aircraft. If the state of the convective weather is w (as section 2), we define a state of the system as $s = (w, X^1, \ldots, X^N) \in \mathbf{R}^{2N+1}$, which represents the positions of all the aircraft and the storm situation . Furthermore, we define S as the set of all possible states s. At any stage t, we want to choose an action (the directions of all the N aircraft to follow) from the set of allowable actions in state s, A_s. Let, $A = \cup_{s \in S} A_s$. We assume that S and A do not vary with time.

If we decide to choose an action (the directions of all the N aircraft to follow) $a \in A_s$ in state s at the stage t, we pay a cost $c_t(s, a)$, which is the sum of distances travelled by N aircraft with the action a. For notational simplicity, we assume that the velocities of all the aircraft are equal (v_{AC}). Hence, if we minimize the expected distance travelled, we are minimize the expected delay. (Exactly the same optimization formulation holds even if the velocities of the aircraft are different from each other: where we need to multiply the cost functions with appropriate multiplying factors). Furthermore, we define an indicator function $I_{kj}(X_k)$, whose value is 1 if X_k is in the sector j, and 0 otherwise. In our optimization problem, we will like to minimize the expected sum of the distances travelled by N aircraft, while resolving all the potential conflicts and satisfying the sector capacity constraints. The optimization problem can be written as,

$$\min_{a\in A} \mathbf{E}_s \left(\sum_{t=T}^{nT} c_t(s,a)\right)$$
$$\text{s.t.} \|X^i(t,w) - X^{j\neq i}(t,w)\|_2 > r,\ \forall i,j\ \forall\ t \forall\ w,$$
$$\sum_k I_{kj} \leq C_j\ \forall\ 0 \leq j \leq f,$$

where r is the minimum permissible separation distance between two aircraft, and $\|.\|_2$ is the euclidian norm [1].

Scheme 2: (priority of aircraft 1)>(priority of aircraft 2)> ... >(priority of aircraft N)

This is a sequential optimization problem and the steps are as follows,
Step 1: In this step, we assume that there is only one aircraft in the airspace and that is aircraft 1, which has the highest priority. The state of the optimization problem is defined as $s_1 = (w, X^1) \in S_1$, where w is the storm state and X^1 is the position of the aircraft 1. If we decide to choose an action $a_1 \in A_{s1}$ in state s_1 at the stage t, we pay a cost $c_t^1(s_1, a_1)$, which is the distance travelled by aircraft 1 with the action a_1. We optimize the following problem,

$$\min_{a_1\in A_{s1}} \mathbf{E}_{s_1} \left(\sum_{t=T}^{nT} c_t^1(s_1,a_1)\right)$$

If we solve this optimization problem, we obtain the optimal policy a_1^o which provides us $X_{opt}^1(t,w)$, the optimal position of aircraft 1 at time t and the storm state w.
Step 2: In this step, we only consider aircraft 2, which has the second highest priority. The state of the optimization problem $s_2 = (w, X^2) \in S_2$. At any stage t, we want to choose an action (the direction aircraft 2 to follow) from the set of allowable actions in state s_2, A_{s2}. Let, $A_2 = \cup_{s_2\in S_2} A_{s2}$. For the aircraft 2, we solve the following optimization problem,

$$\min_{a_2\in A_{s2}} \mathbf{E}_{s_2} \left(\sum_{t=T}^{nT} c_t^2(s_2,a_2)\right)$$
$$\|X^2(t,w) - X_{opt}^1(t,w)\|_2 > r,\ \forall\ 0 \leq t \leq nT\ \forall\ w,$$

and obtain $X_{opt}^2(t,w)$.
Step 3- Step N: We keep following the same procedure till we have solved for all N aircraft. For the Nth aircraft,which has the lowest priority, the optimization problem is the following,

$$\min_{a_N\in A_{sN}} \mathbf{E}_{s_N} \left(\sum_{t=T}^{nT} c_t^N(s_N,a_N)\right)$$
$$\text{s.t.} \|X^N(t,w) - X_{opt}^1(t,w)\|_2 > r,\ \forall\ t\ \forall\ w,$$
$$\|X^N(t,w) - X_{opt}^2(t,w)\|_2 > r,\ \forall\ t\ \forall\ w,$$
$$\ldots$$
$$\|X^N(t,w) - X_{opt}^{(N-1)}(t,w)\|_2 > r,\ \forall\ t\ \forall\ w,$$
$$\sum_k I_{kj} \leq C_j\ \forall\ 0 \leq j \leq f,$$

where $X_{opt}^1, \ldots, X_{opt}^{(N-1)}$ are obtained from previous iterations.

In both of the schemes, we look for the "best policy". Determining the "best policy" is to decide where to go next given the currently available information. We consider the set of decisions facing all of the aircraft that start moving towards the destination along a certain path, with the recourse option of choosing a new path whenever a new information is obtained.

4 Markov Decision Process

Finite-state and finite-action Markov Decision Processes (MDPs) capture several attractive features that are important in decision-making under uncertainty: they handle risk in sequential decision-making via a state transition probability matrix, while taking into account the possibility of information gathering and recourse corresponding to this information during the multi-stage decision process [12, 2]. The Markov decision process has the finite state $\mathcal{X}$, and the finite action set $\mathcal{A}$. We denote by $P = (P^a)_{a\in\mathcal{A}}$ the collection of transition matrices, and by $c_t(i,a)$ the cost corresponding to state i and action a at time t, and denote by c_T the cost function at the terminal time, T.

The optimization problem is to minimize the

[1] If $M = (M_1, M_2, \ldots, M_p) \in \mathbf{R}^p$, then $\|M\|_2 = \sqrt{M_1^2 + \ldots + M_p^2}$.

expected cost over a finite horizon:

$$\min_{a\in A} \mathbf{E}\left(\sum_{t=0}^{T-1} c_t(i_t,a_t)+c_T(i_T)\right)$$

where $a=(a_0,\ldots,a_{T-1})$ denotes the strategy and A the strategy space. When the transition matrices are known, the value function can be computed via the Bellman recursion

$$V_t(i)=\min_{a\in\mathcal{A}}\left(c_t(i,a)+\sum_j P^a(i,j)V_{t+1}(j)\right).$$

5 Solution of Traffic Flow Management where all of the N aircraft has the equal priority (scheme 1)

We propose a Markov Decision Process algorithm to solve the traffic flow management where each of the N aircraft has same priority. The steps of the algorithm are as follows,

Step 1: Preliminary calculations

The state of the Markov Decision Process (MDP) for this problem is $s=(w,X^1,X^2,\ldots,X^N)$ ($(2N+1)$ tuple vector) and $s\in S$ (defined in the section 3). If we discritize the airspace by D number of nodes, $|S|=D^{2N}(l+1)^m$. There are n stages in this MDP (obtained in section 2). In addition, we need to calculate the action set of the MDP. Actions of this MDP are the directions of all the N aircraft to follow in each stage with different realizations of the weather. We can obtain the action a, that is the directions of the aircraft to follow if we calculate the points that can be reached by each aircraft in the next 15 minutes, given their current positions. This can be approximately calculated if we draw an annular region with $15\times v_{AC}\pm\epsilon$ as radii, with a predefined angle θ and checking which grid points fall in the region. Once we have the set of all possible controls, we readily obtain P^a from the weather data.

Step 2: Assigning appropriate costs

We assign costs in such a way that our algorithm provides paths that include going through the zones in the absence of storms while avoid it if there is a storm. Furthermore, it should also make sure that there is no conflict among N aircraft. We define $c(w,X^1,\ldots,X^N,(X^{1i}),\ldots,(X^{Ni}))$ as the sum of all $1\le k\le N$ costs obtained by aircraft k to go to X^{Ki} from X^k.

Provision 1: Avoid if storm, otherwise take a shortcut

We introduce a function $PROV_1:\mathbf{R}^{4N+1}\rightarrow\{0,1\}$ in order to provide us the provision of avoiding a zone if there is a storm, otherwise taking a shortcut.

If $\forall 1\le k\le N$, ($X^{ki}\in\{k_1 or k_2 or..k_m\}$) or ({the line segment ($\lambda X^{ki}+(1-\lambda)X^k$ and $0\le\lambda\le 1$) connecting the points X^k, X^{ki} cut any of the predicted storm zone } & { the storm state w corresponds to a storm at that particular zone})

$PROV_1(w,X^1,\ldots,X^N,X^{1i},\ldots,X^{Ni})=1$,

else

$PROV_1(w,X^1,\ldots,X^N,X^{1i},\ldots,X^{Ni})=0$

endif.

Provision 2: Avoid conflict among each other

First, we introduce a function $CF_{v_1v_2}:\mathbf{R}^2\times\mathbf{R}^2\times\mathbf{R}^2\times\mathbf{R}^2\rightarrow\{0,1\}$, where it takes the origin and destination points of two aircraft with velocities v_1 and v_2 and provides "$1''$ if they are in conflict and "$0''$ otherwise. We will demonstrate how to obtain $CF_{v_1v_2}(I_1,F_1,I_2,F_2)$, where I_j is the initial point and F_j is the final point of the aircraft j. At time t, the positions of aircraft 1 and 2 are $I_1+\frac{F_1-I_1}{\|F_1-I_1\|_2}v_1t$ and $I_2+\frac{F_2-I_2}{\|F_2-I_2\|_2}v_2t$ respectively. The distance between them at time t, $d(t)=\|\Delta I-(\Delta W)t\|_2$, where $U_1=\frac{F_1-I_1}{\|F_1-I_1\|_2}$, $U_1=\frac{F_2-I_2}{\|F_2-I_2\|_2}$, $\Delta I=I_2-I_1$, and $\Delta W=(v_1U_1-v_2U_2)$. $\arg\min_t d(t)=\arg\min_t(\Delta I-t\Delta W)^T(\Delta I-t\Delta W)$. In order to find the optimal time t^* at which two aircraft come to the closest point, we set $\frac{\partial}{\partial t}(\Delta I-\Delta Wt)^T(\Delta I-\Delta Wt)=0$. Solving this, we obtain $t^*=\frac{\Delta W^T\Delta I}{\Delta W^T\Delta W}$. If $t^*<0$, the two aircraft are diverging, hence $CF_{v_1v_2}(I_1,F_1,I_2,F_2)=0$. Also, if $\|\Delta I+t^*\Delta W\|_2>r$, $CF_{v_1v_2}(I_1,F_1,I_2,F_2)=0$, else $CF_{v_1v_2}(I_1,F_1,I_2,F_2)=1$.

In this problem, as we have assumed that all the aircraft are flying at the same speed, we can write $CF(.,.,.,.)$ instead of $CF_{v_1v_2}(.,.,.,.)$. In addition, we introduce a function $PROV_2:\mathbf{R}^{4N}\rightarrow\{0,1\}$ that provides us the provision of conflict avoidance.

If $\forall l,k$, $\|X^{li}-X^{ki}\|_2 > r$ & $CF(X^l, X^{li}, X^k, X^{ki}) = 0$, $PROV_2(X^1,\dots,X^N,X^{1i},\dots,X^{Ni}) = 0$,
else
$PROV_2(X^1,\dots,X^N,X^{1i},\dots,X^{Ni}) = 1$
endif.
Finally, the cost function is defined as following,
if $PROV_1(w,X^1,\dots,X^N,X^{1i},\dots,X^{Ni}) = 1$ & $PROV_2(X^1,\dots,X^N,X^{1i},\dots,X^{Ni}) = 1$,
$c(w,X^1,\dots,X^N,X^{1i},\dots,X^{Ni}) = \infty$,
else
$c(w,X^1,\dots,X^N,X^{1i},\dots,X^{Ni}) = \sum_{k=1}^{N} \|X^{ki} - X^k\|_2$
endif.

Step 3: Assigning appropriate Value function

We define $V_t(s)$ as the value function which is the expected minimum distance to go if the current state is s and the current stage is t. We need to add the following provisions in the value function in order to obtain the correct solution.

Provision 1: Reach the destination points

The value function should have boundary conditions such that we obtain a complete path (path stating at the origin and ending at the destination) as a solution. For the destination points $D^1,\dots,D^N$, the conditions below would guarantee that the solution will provide a complete path. For any state weather state w and the last stage n,
if $\{X^1 = D^1\},\dots,\{X^1 = D^1\}$
$V_n(w,X^1,\dots,X^N) = 0$,
else
$V_n(w,X^1,\dots,X^N) = \infty$
endif.

Provision 2: Direct cost at the end of the flight

We assign the boundary values to the value function for the states which corresponds to the aircraft locations that are less than $15v_{AC}$ apart from the destination points. Let the state corresponds to the aircraft location of $X^1,\dots,X^N$ and $(\|(X^i - D^i\|_2 \leq 15v_{AC}\ \forall i)$ and the stage $t > 1$.
If (w corresponds to the storm state such that no storm zone intersects the straight line $\{\lambda X^k + (1-\lambda)D^k$ and $0 \leq \lambda \leq 1\}$)
$V_t(w,X^1,\dots,X^N) = \sum_k \|X^k - D^k\|_2$, for any $t > 1$,
else
$V_t(w,X^1,\dots,X^N) = \infty$
endif.

Provision 3: Sector Capacity

In order to ensure that the total number of aircraft in a sector at any time does not exceed the sector capacity, we assign value function appropriately. For a state $s = (w,X^1,\dots,X^N)$, if there exists at least one j such that $\sum_k I_{kj} > Cj$, then $V_t = \infty$.

Step 4: Implementing the recursive equations

The recursive equation that solves the problem is as follows,

$$V_t(s) = \min_{a\in A}\{c(s,a) + \sum_{s'} P^a(s,s')V_{t+1}(s')\}.$$

We use the backward dynamic programming technique to solve these equations. We start with the final stage and go back iteratively to the first stage and obtain solutions for every stage and for every state. At the first stage, the solution is readily obtained as we know the current state. The aircraft will keep continue flying according to the solution till a new update is obtained. At the next stage, we will receive a new update, which corresponds to a new state. As we have already calculated all the optimal control for all possible states, we just check the vector $V_2(.)$ and obtain the control. The aircraft will proceed in this way till they reach the destination points (checking the vector $V_n(.)$). In this way, we compute a routing strategy that provides the minimum expected delay.

6 Solution of Traffic Flow Management with different priority (scheme 2)

In this scheme, we assume that (priority of aircraft 1)>(priority of aircraft 2)> ... >(priority of aircraft N) (as described in the section 3).

Step 1: Optimal route for aircraft 1

In this step, we find the optimal route for aircraft 1, which has the highest priority. In a sense, we assume that there is no aircraft in the airspace.

The MDP state $s_1 = (w, X^1) \in \mathbf{R}^3$ and $s_1 \in S_1$ (defined in the section 3). We discritize the airspace and time in a same way as described in section 5 and section 2, which yields $|S_1| = D^2(l+1)^m$, and n stages in this MDP. Actions of this MDP are the directions of aircraft 1 to follow in each stage with different realizations of the weather. As described in section 5, we obtain the action a_1, that is the directions of aircraft 1 to follow if we calculate the points that can be reached by aircraft 1 in the next 15 minutes, given its current position. We define $c_1(w, X^1, X^{1i})$ as the cost to go if the aircraft 1 goes from X^1 to X^{1i} in a stage. For assigning the appropriate value, we only need to add provision 1 (calculation of which is same as described in section 5: step 2: provision 1). As there is only one aircraft, there is no need to add the provision for conflict avoidance. The value function for the MDP is defined as $V_t^1(s_1)$, which is the expected minimum distance to go if the current state is s_1 and the current stage is t. In the value function, we add the the first two provisions (the calculation is same as described in section 5: step 4: provision 1and 2). As there is no other aircraft in the airspace, we do not need to add any provision that require satisfying the sector capacities. Once we have assigned all the boundary values properly, we can solve the following recursion,

$$V_t^1(s_1) = \min_{a_1 \in A_1} \{c_1(s_1, a_1) + \sum_{s_1'} P^a(s_1, s_1') V_{t+1}^1(s_1')\},$$

and we obtain the solution of the recursion which provides us $X_{opt}^1(t, w)$.

Step $2-N$: Optimal route for aircraft $2-N$

We follow the same procedure in the next steps, except we add the provisions that prohibit conflicts and satisfy the sector capacity constraints. In the the second step, we add the conflict avoidance provision in the cost function, where $X_{opt}^1(t, w)\ \forall\ t\ \forall w$ are considered "NO-GO" zones. We use the same procedure described in section 5: step 3: provision 2, where we assign high cost for violating these constraints. In each iteration, we keep a record of I_{kj} and assign a very a high value to the value function if the sector capacity constraints are violated. When we solve the appropriate recursion, these additional features will guarantee the conflict avoidance and satisfy the sector capacity constraints. For the Nth aircraft, which has the lowest priority, the provision 2 for cost function will be as follows, if $\forall 1 \le k \le N-1$, $\|X^{Ni} - X^{ki}\|_2 > r$ & $CF(X^N, X^{Ni}, X_{opt}^k, X_{opt}^{ki}) = 0$,
$PROV_2^N(X^N, X^{Ni}) = 0$,
else
$PROV_2^N(X^N, X^{Ni}) = 1$
endif.
The cost function will be defined in the same way (section 5) using these two provisions. Also, $V_t^N(.)$ is very high for the states which violates the sector capacity constraints. With this provisions in the cost and value functions, we can solve the recursion and obtain optimal strategy for all of the N aircraft that minimize the delay, given the above priority scheme. This scheme avoids combinatorial explosion as $|S_1 = |S_2| = \ldots = |S_N| = D^2(l+1)^m$. On the other hand, as this is a more constrained optimization problem, it yields higher delays than the scheme 1.

7 Various applications in the European airspace

In the European airspace, weather is not as acute of a problem as in US. However, the algorithms presented here can be applied to some important problems that arise in Europe as well.
Routing under congested airspace: Stochastic obstacles that occur frequently in Europe is the congested airspace. The dynamics of the congested airspace is stochastic in nature. The remaining capacity of the airspace to be used by the aircraft outside the congested airspace can be expressed in a Markovian model. Our proposed MDP algorithm can be readily applied to the problem involving the capacity uncertainty due to congestion.
Routing of platoon of aircraft under uncertainty: In Europe, multiple aircraft fly in a platoon to go to a same destination point, where priorities are given according to the position in the platoon; i.e., the leading aircraft getting the highest priority and the second in line getting the second highest priority and so on. We can apply our algorithm (scheme 2) in order to obtain the optimal routing strategy of the aircraft. The solution will also yield the diverging (when the aircraft get separated) and converging (when they merge again)

points of the platoon. This is actually a special case of the algorithm proposed by us where all the destination points are same.

8 Simulation

In this section, we discus the results of the implementation of both of our algorithms in various scenarios involving dynamic routing and traffic flow management of multiple aircraft under uncertainty.

We have implemented both of the algorithms in MATLAB and we ran our experiment on a standard PC. In the first experiment, there are two aircraft with origins at $O^1 = [0, 96]^T$, and $O^2 = [0, -96]^T$, and destinations at $D^1 = [312, -96]^T$ and $D^2 = [312, 96]^T$ (all the units in n.mi.). The velocities of the aircraft are 480 n.mi/hour. There is a prediction of a convective weather. The storm zone is a rectangle whose corner points are $[168, 96]^T$, $[168, -96]^T$, $[192, -96]^T$, $[192, 96]^T$, which may obstruct the nominal flight path of the aircraft. Moreover, there is a critical airspace within this zone which will definitely be affected if the the storm takes place. The critical zone is assumed a rectangle with corner points $[168, 60]^T$, $[168, -60]^T$, $[192, -60]^T$, $[192, 60]^T$ (the shaded zone in figure 5). We assume that the weather information of the portion of the airspace that can be reached in 15 minutes is deterministic and the probability of the storm propagates in a Markovian fashion with time. Also, the minimum separation distance between two aircraft, $r = 5$(n.mi) in this example.

The weather update is received once every 15 minutes. We discritize the time in 15 minutes time intervals (stages). We define "0" as the state when there is no storm, "1" as the state when only the critical zone is affected by the storm, and "2" as the state when the whole predicted zone has been affected by the storm. The prediction matrix is a follows,

$$P = \begin{pmatrix} 0.4 & 0.4 & 0.2 \\ 0.3 & 0.3 & 0.3 \\ 0.3 & 0.3 & 0.3 \end{pmatrix}.$$

$P(i+1, j+1)$ corresponds to the probability that the storm state will be j in the next stage, if the current storm state is i; i.e., $P(2, 1) = 0.3$ means that the probability that the storm state will be 1 (no storm) in the next stage is 0.3 given the current storm state is 0 (only critical zone is affected by the storm).

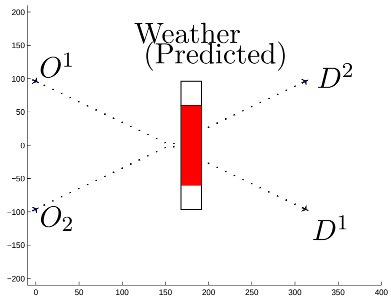

Figure 5: Routing of two aircraft.

In this scenario, we implemented both of our strategies (scheme 1 and 2) and compared their performances with the traditional strategy (TS) where the convective zone is avoided as if it is a deterministic obstacle while avoiding conflicts. We define " Delay Measure (DM_i^A)" as the extra flight path required by aircraft i in a strategy A in excess of the nominal flight path (which is the Euclidean distance between the origin and the destination; 366.34 (n.mi.) in this problem); $DM_{ij}^A = DM_i^A + DM_j^A$. Furthermore, we introduce a performance metric "Improvement Measure ($IM_i^{A/B}$)" of Strategy 'A' over 'B', which is the percentage of the maximum possible improvement gained for aircraft i by using strategy 'A' instead of using strategy 'B'; $IM_i^{A/B} = 100 \times \frac{DM_i^B - DM_i^A}{DM_i^B}$, and $IM_{ij}^{A/B} = 100 \times \frac{DM_{ij}^B - DM_{ij}^A}{DM_{ij}^B}$. Higher IM corresponds to better delay. In TS, if we resolve the conflict, aircraft 1 and 2 follow paths with a length of 455.12n.mi). $DM_1^{TS} = DM_2^{TS} = 455.12 - 366.34 = 88.78$, and $DM_{12}^{TS} = DM_1^{TS} + DM_2^{TS} = 177.56$.

Using the scheme 1, where both aircraft have equal priority, aircraft 1 and 2 will initially follow a path with an angle of 30.96^0 and -30.96^0 respectively till they get the next update. Both of them will avoid the storm zone when there is a storm and take a direct route if there is no storm and the solution of the strategy is conflict free. In this way, both the aircraft follow a flight path that yield a expected delay of 398.67 (n.mi.).

$DM_1^1 = DM_2^1 = 32.33$, $IM_1^{1/TS} = IM_2^{1/TS} = IM_{12}^{1/TS} = 100 \times \frac{88.78-32.33}{74.78} = 63.58\%$. Similarly if we use scheme 2, where aircraft 1 has higher priority over the aircraft 2, aircraft 1 and 2 initially fly at an angle -36.86^0 and 53.13^0 till they get the next update. Similar to the scheme 1, both of them will avoid the storm zone when there is a storm and take a direct route if there is no storm and the solution of the strategy is conflict free. The expected distance travelled by the aircraft are 382.08 (n.mi.) and 426.24(n.mi.) respectively; $IM_1^{2/TS} = 78.95\%$, $IM_2^{2/TS} = 40.18\%$, and $IM_{12}^{2/TS} = 56.76\%$. The summary of the result is presented in 1.

	IM of Scheme 1 over TS	IM of Scheme 2 over TS
Aircraft 1	56.71%	78.95 %
Aircraft 2	67.72%	40.18%
System	63.03 %	56.76%

Table 1: Improvement comparisons

We observe that we obtain a better system delay in case of scheme 1. However, in real life, depending upon the aircraft type, size, and hub and spoke network, it might be more reasonable to prioritize the routing strategy. Moreover, the computation time for scheme 2 is 8.31 seconds, which is much faster than the computation time for scheme 1 (7.46 minutes). We can avoid a combinatorial explosion in case of scheme 2 and can handle large number of aircraft. There is no significant computation cost in adding an extra aircraft. In order to illustrate this point, if we add one more aircraft in the system with $O^3 = [0,0]^T$ and $D^3 = [360,0]^T$ (figure 6), our algorithm gives the routing strategy for aircraft 3 with an additional 4.84 secs. Aircraft 3 should have an initial angle of 14.036^0, which yields $IM_3^{2/TS} = 34.62\%$ (for aircraft 3) and $IM_{123}^{2/TS} = 51.23\%$ (for the system).

In addition, we ran an experiment for the same weather prediction where a platoon of aircraft are positioned at $O^1 = [48,48]^T$, $O^2 = [24,24]^T$, and $O^3 = [0,0]^T$. The destination point for all of the them is $[360, 0^T]$ and (priority of aircraft 1)>(priority of aircraft 2)>(priority of aircraft 3). Initial vector provided by our algorithms for the aircraft are 33.61^0, 36.65^0, and 38.65^0. The system level IM obtained by using scheme 2 over TS

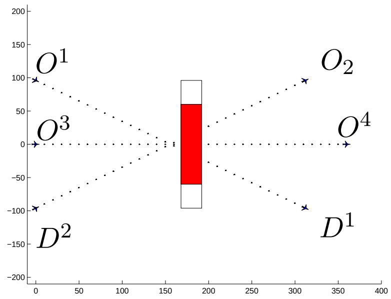

Figure 6: Routing of three aircraft.

is 58.35% (figure 7).

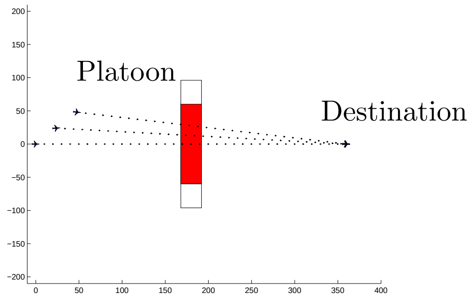

Figure 7: Routing of a platoon of aircraft.

9 Conclusion

In this paper, we provide a traffic flow management tool that can be used by Air Traffic Controller or Airline Dispatcher to dynamically route multiple aircraft under uncertain weather. Our solution provides a routing strategy for aircraft which minimizes the expected delay of the system. Moreover, as it provides less circuitous routes, it inhibits the overloading of aircraft in the neighboring sectors of the predicted storm zones. Consequently, it restricts the ripple effect of delay due to convective weather.

The complexity of the computation depends on the number of the aircraft, the origin-destination pair, size and location of the storms, type of storm, level of discretization, and the rate of information updates. The complexity of the algorithm, when all the aircraft have equal priority, does not scale well with the number of aircraft. However, when each aircraft has different priorities, the algorithm is scalable; i.e., the algorithm scales polynomially with the number of aircraft. In real life, a mixed scheme is more appropriate; aircraft are prioritized in a number of sets such that aircraft in different sets have the different priorities, but the aircraft within each set have the same priority. Our algorithm can be readily applied to the mixed scheme.

Currently, we are developing a priority scheme based on the game theoretic model that will be fair to all the participants in the system. In addition, we are working on proposing an approximate dynamic programming model that can handle large scale problems involving a large number of aircraft and convective zones.

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